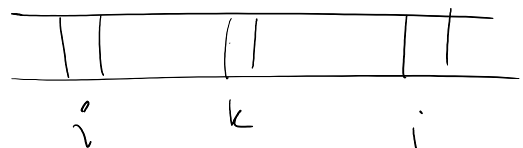
Fine grained complexity 2018/19

10 January, 2019 23:13

monotonicity testing

Ailon, Chazelle, Seshadri, + ≈ 2005

- i is heavy iff $\exists j$



s.t. for $1/3$ of elements k between i & j

$$f(k) < f(i)$$

(similarly the other way around

$f(k) > f(i)$)

Claim: If $f : [n] \to \mathbb{R}$ requires εn places to modify to be monotone then

(1) there are at least εn i's that are heavy

(2) there are at most $3\varepsilon n$ i's that are heavy

each place that needs to be corrected

$$f(i) > f(j)$$

either i is heavy or j is heavy

removing all heavy items makes f monotone removing all heavy items makes f monotone

mark places that suffice to remove to make f monotone

C

i ~ heavy ~ j

for each right-heavy $i \notin C$ there are 1/3 elts between i & j that belong to C. Assign i to any one of them that has fewer than 2 of such i's assigned so far.

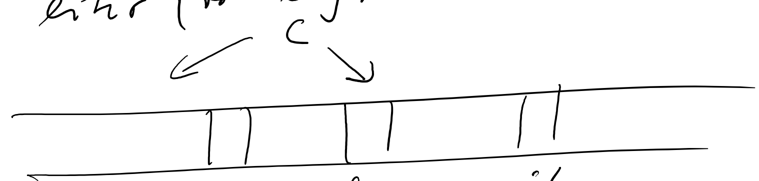
going from right -> left

No elt in C gets assigned to more than two heavy elts not in C.

Repeat the same for heavy from left.

(Each elt in C gets stuff assigned either for left or right!

C



$i > f(k) > i'$

$\notin C$

$\Rightarrow f(i) > f(i')$

i.e. f not monotone after removing C.

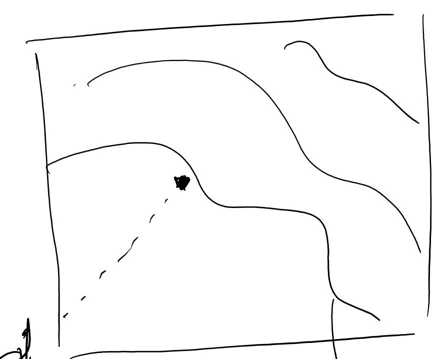
after reading C.

- Can test the claim efficiently using $\tilde{O}(\lg^3 n)$ samples.

$p \ldots$ prime $\approx n^3$ $\quad a \in_R \mathbb{F}_p$

x, y $\quad r(x_{1..k}) = \sum_{i=1}^{k} x_i a^i$

$O(n+k^2)$
e.d. algorithm



slide

d

$F^h(d)$ = the furthest pt along the diagonal of edit distance at most h.